THE EFFICIENCY OF THE MOTION AMPLIFICATION DEVICE WITH VISCOUS DAMPER

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ABSTRACT

The motion amplification device with a viscous damper has been recognized as an effective solution to mitigate wind or seismic excitation especially for stiff-type structural systems. These devices are designed to amplify a small interstory drift to amplify the stroke of dampers attached. The efficiency of such devices with dampers relies not only on geometric configurations but is highly dependent on the stiffness of support elements. In this paper, a "scissor-jack" type of motion amplification device, a "toggle brace damper" system, is investigated. A procedure for determining the relationship between the motion amplification factors with geometry of the toggle brace mechanism, which includes the elongation of the braces is proposed. It is demonstrated that the amplification factor is not merely a function of toggle brace configuration; it also depends on the brace stiffness, including toggle brace elongation. Accordingly, a mathematic model in the complex modulus of the toggle brace damper system is established and results are presented. The analysis results indicate that the efficiency of the toggle brace damper system significantly depends on support toggle brace stiffness, and this is an important design consideration.

Introduction

Viscous dampers have proved to be a very efficient device to absorb and dissipate large amounts of energy from earthquake or wind in order to maintain the structural response within acceptable limits. These devices are ideally suited for flexible structures. Recent efforts were dedicated to find methods to improve the efficiency of the dampers to stiff structures. Various motion amplification devices have been discussed by Hanson and Soong (2001) to amplify small deflections, which may render viscous damping device ineffective. The "scissor-jack" type of motion amplification devices, so called "toggle brace damper", has been proposed and patented by Taylor in 1996. Constantinou, et. al. (2001) demonstrated that the toggle brace damper system can improve the efficiency of this energy dissipation device by magnifying the damper displacement and also verified its ability to enhance the efficiency of the toggle brace damper system through both cyclic loading tests and shaking table tests with a single-degree-of-freedom steel frame. McNamara and Huang (2000) adopted this concept, applied the toggle brace damper system to a 39-story office building in Boston, and was completed in 2000. Their computer analysis showed that the stiffness of the toggle braces played a very important role for enhancing the effectiveness of the overall system damper. In that application, they revised the attachment of

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the lower brace proposed by Constantinou, et. al. (2001) and installed it directly into the beam-column joints, which eliminated the deflection from the beam. Hwang, et.al (2005) proved that this modification was necessary to improve the effectiveness of the toggle brace damper system from his theoretical analysis and his laboratory tests. Shao and Miyamoto (2002) also used the toggle brace damper to retrofit the torsional irregularity for stiff concrete shear wall structures to achieve the Enhanced Rehabilitation Objective of FEMA 356. Berton, et. al. (2004) proposed the displacement amplification device constructed using a combination of two rack and pinion mechanisms. From his labor tests, no discussion of the effectiveness of the damper was made. Ribakov and Reinhorn (2003) proposed an optimized solution for the amplified structural damping. The amplification factors in both papers used the concept proposed by Constantinou, et. al. (2001), which was the function of the toggle brace configurations. However, this may result in overestimating amplification effect, including the toggle brace stiffness and reduce the overall effectiveness of viscous damper attached. Huang (2004) reestablished equilibrium equations and compatibility relationships by incorporating the toggle brace elongations in a series of coupling equations based on the simple static relationship assumptions. The coupling equations can be solved by using linear programming method. Parametric studies are also given according to the variation of the story drifts, damping coefficients, toggle brace stiffness, etc. In this paper, a simple constitutive relationship of the toggle brace damper system in complex modulus form is established for harmonic excitation instead of static relationship assumption. The analysis results show that the efficiency of the toggle brace damper system is highly dependant on the toggle brace stiffness along with the damper damping value. Compared with other bending types of motion amplification devices, the toggle brace damper proved to be one of the most effective devices from the lever mechanism because stiffer properties can be provided.

Amplification Factors of Toggle Brace Damper Systems

The revised lower toggle brace damper configuration proposed by McNamara and Huang (2000) is illustrated in Figure 1. The attachment of damper in the toggle brace damper system directly connected to the beam-column joint rather than the beam, because the deformation of the beam due to damper force exerted on will reduce the effective damping contributed by the damper. Also, it will significantly affect the design of the floor beam.

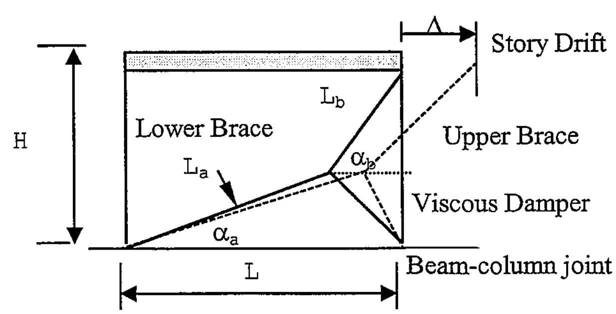

Figure 1 - Toggle Brace Damper (TBD) System Configuration

The geometric relation between the deformed and undeformed frame was established by Huang (2004) according to its horizontal and vertical direction geometric relation below,

$$L_a(1+\varepsilon_a)\cos(\alpha'_a)+L_b(1+\varepsilon_b)\cos(\alpha'_b)=L+\Delta \qquad \text{Eq. 1}$$

$$L_a(1+\varepsilon_a)\sin(\alpha'_a) + L_b(1+\varepsilon_b)\sin(\alpha'_b) = \tan(\alpha)L \qquad \text{Eq. 2}$$

where Δ is the story drift; L is the frame bay length and H is the interstory height. Further, the ratio of the story height and frame bay length can be defined by $\alpha = tan^{-1}(H/L)$. L_a and L_b are the length of lower and upper toggle braces. ε_a and ε_b are the strains of lower and upper brace members with elongation ($\varepsilon = \Delta L/L$). α_a' and α_b' are the horizontal angles of lower and upper braces after frame distortion with prime symbol. The equilibrium equations at pivot joint can be written according to horizontal and vertical directions

$$EA_a\varepsilon_a\cos(\alpha'_a) - EA_b\varepsilon_b\cos(\alpha'_b) = FD\cos(\alpha'_c) \qquad \text{Eq. 3}$$

$$-EA_a\varepsilon_a\sin(\alpha'_a) + EA_b\varepsilon_b\sin(\alpha'_b) = FD\sin(\alpha'_c) \qquad \text{Eq. 4}$$

where the damper force (FD) can be defined as the linear function of the velocity $FD = c\,v$ and c is the damper coefficient, v is the relative velocity of the damper. A_a and A_b are the cross-section area of lower and upper brace member and E is the modulus of the braces member. α_c' is the angle of the damper to horizontal beam after frame deformed. To simplify the equation, we have the simple geometric relation shown as

$$\frac{L_a(1+\varepsilon_a)}{\sin(\alpha'_c)} = \frac{L}{\sin(\alpha'_c + \alpha'_a)} \qquad \text{Eq. 5}$$

Relative displacement (δ_c) or the deformation of the damper is calculated by the difference of the damper displacement before and after frame deformed

$$\delta_c = \sqrt{[L_a(1+\varepsilon_a)]^2 + L^2 - 2L_a(1+\varepsilon_a)L\cos(\alpha'_a)} - \sqrt{L_a^{\,2} + L^2 - 2L_a.} \qquad \text{Eq. 6}$$

where α_a, and α_b are the horizontal angle of the lower brace and upper brace angle before the frame deformed. The relative velocity (v) of the damper can be calculated,

$$v = \frac{d}{dt}(\delta_c(\Delta)) = \frac{d\delta_c(\Delta)}{d\Delta}\frac{d\Delta}{dt} \qquad \text{Eq. 7}$$

Assuming a steady harmonic excitation with radian frequency (ω) is applied at floor level, the relative velocity of the viscous damper in Eq. 7 can be simplified by $v = Am(\Delta)\ \omega\ \Delta$ where the amplification factor $Am(\Delta) = d\delta_s(\Delta)/d\Delta$. The damper forces $FD = c\ Am(\Delta)\ \omega\ \Delta$ for linear velocity-related damper.

For given frame bay length (L), story height (H), the lengths of the lower and upper brace members (L_a, L_b) and brace section properties (E, A_a and A_b), damping constant (c), the six unknown variables α_a' α_b' α_c' ε_a ε_b and Am can be solved by combining Equations 1 through 6 as the function of the story drift (Δ). The axial forces of lower brace (FA) and upper brace (FB) are simply derived in terms of the damper force (FD) from Eqs.3 and 4,

$$FA = FD\frac{\sin(\alpha'_c + \alpha'_b)}{\sin(\alpha'_b - \alpha'_a)}; \qquad FB = FD\frac{\sin(\alpha'_c + \alpha'_a)}{\sin(\alpha'_b - \alpha'_a)} \qquad \text{Eq. 8}$$

The damper displacement or stroke corresponding to story drift comparison with and without, including the elongation of the braces, is shown in Figure 2. The amplification factor *($\delta c/\Delta$)* keeps constant with the brace stiffness. Particularly, the amplification factor is reduced in significant amount as story drift gets larger. Additional discussions were given by Huang (2004) for other design parameters, such as brace stiffness and damping value.

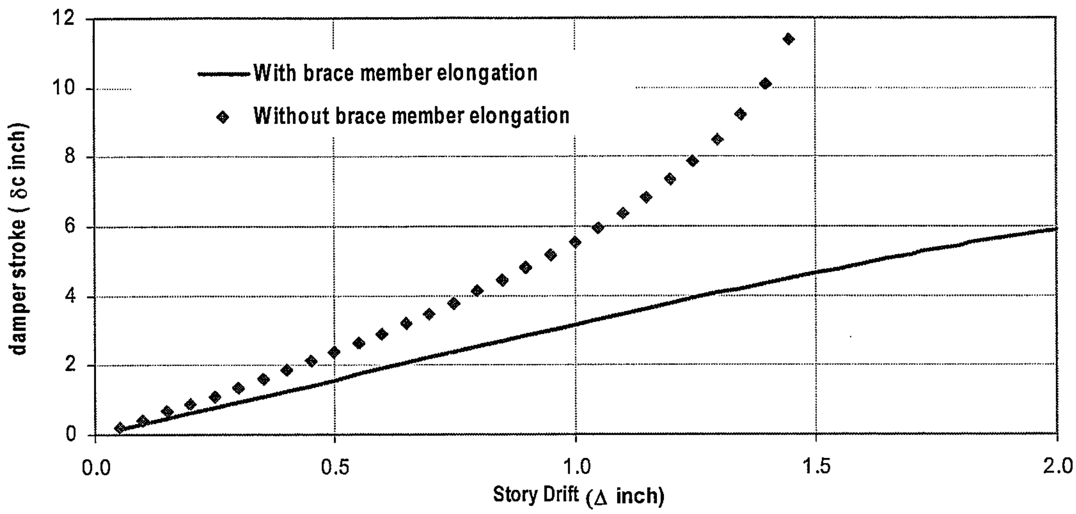

Figure 2 - The Displacement of Damper vs. Story Drift

Considering a single-degree-of-freedom (SDOF) dynamic system, and define $x = \Delta(t)$, the basic differential motion equation with mass (m) can be expressed by,

$$m\ddot{x} + f(x,\dot{x}) = g(t) \qquad \text{Eq. 9}$$

Applied force g(t) can be wind force or earthquake excitation, ω is the natural radian frequency of structure. For linear damping, the restore force is,

$$f(x,\dot{x}) = cAmf(x)\dot{x} + m\omega^2 x \qquad \text{Eq. 10}$$

Where the force amplification factor *Amf* as a function of story drift (x) can be written,

$$Amf(x) = \frac{d\delta_c(x)}{dx}\frac{\sin(\alpha_c' + \alpha_a')}{\sin(\alpha_b' - \alpha_a')}\cos(\alpha_b') \qquad \text{Eq. 11}$$

From above derivation, no small story drift is required. It can be applied to the large deformation of story drift. From Equation 11, the force amplification is combined with displacement amplification and geometry amplification, in other words, the small damper force can exert the large brace forces to resist external lateral forces, so called leverage effect. Therefore, softening the brace stiffness will decrease the amplification effect and further reduce the effectiveness of the damper system.

Complex Modulus of Toggle Brace Damper Systems for Harmonic Excitation

Equations 1 to 6 outline the geometric relationship of the motion amplification and force amplification with story drift by incorporating elongations of the braces. In dynamic response analysis for the toggle brace damper system, dissipated energy and phase angle will directly determine the efficiency of the toggle brace damper system. The toggle brace damper system can be mathematically described as a series of the spring-dashpot model with combination of Maxwell model and Voigt model, (see Figure 3)

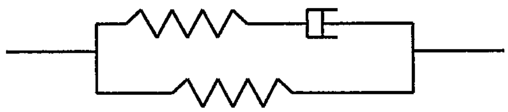

Figure - 3 Mathematical Model for the Toggle Brace Damper

where K1 is the structural stiffness from the story frame, K2 is the toggle brace stiffness, and C2 is the damping value of the damper. Differentiate Equations 1, 2, and 6 as a function of the story drift, and simplify with Equation 5. We will have the following relation.

$$\frac{\sin(\alpha_c'+\alpha_b')}{\sin(\alpha_c'+\alpha_a')}\frac{1}{\cos(\alpha_b')}\frac{L_a d\varepsilon_a}{dt}+\frac{1}{\cos(\alpha_b')}\frac{L_b d\varepsilon_b}{dt}+\frac{\sin(\alpha_b'-\alpha_a')}{\sin(\alpha_c'+\alpha_a')}\frac{1}{\cos(\alpha_b')}\frac{d\delta_c(\Delta)}{dt}=\frac{d\Delta}{dt} \qquad \text{Eq. 18}$$

For the linear spring-damping model, we have the following relationships by their definitions.

$$\frac{L_a d\varepsilon_a}{dt}=\frac{L_a}{EA_a}\frac{dFA}{dt} \qquad \frac{L_b d\varepsilon_b}{dt}=\frac{L_b}{EA_b}\frac{dFB}{dt} \qquad \frac{d\delta_c(\Delta)}{dt}=\frac{FD}{c} \qquad \text{Eq. 19}$$

From Equation18, the toggle brace damper system has the form of Maxwell model. Phase angles will be determined by ratio of damping value to brace stiffness (so called characteristic time). The horizontal components force of the upper brace F = FB cos (α'_b). Since the variation of the motion amplification is invariant to story drift (see Figure 3), assume the relationships of Equation 8 still hold, reorganize Equation 18 with Equation 19, we have,

$$\left[\left(\frac{\sin(\alpha_c'+\alpha_b')}{\sin(\alpha_c'+\alpha_a')}\frac{1}{\cos(\alpha_b')}\right)^2\frac{L_a}{EA_a}+\left(\frac{1}{\cos(\alpha_b')}\right)^2\frac{L_b}{EA_b}\right]\frac{dF}{dt}+\left(\frac{\sin(\alpha_b'-\alpha_a')}{\sin(\alpha_c'+\alpha_a')}\frac{1}{\cos(\alpha_b')}\right)^2\frac{F}{c}=\frac{d\Delta}{dt} \qquad \text{Eq. 20}$$

The generalized brace stiffness K2 and damping value C2 in Equation 18 can be defined by,

$$K2=E\cos^2(\alpha_b')\left[\frac{A_aA_b}{\left(\frac{\sin(\alpha_c'+\alpha_b')}{\sin(\alpha_c'+\alpha_a')}\right)^2 A_bL_a+A_aL_b}\right] \qquad C2=c\cos^2(\alpha_b')\left(\frac{\sin(\alpha_c'+\alpha_a')}{\sin(\alpha_b'-\alpha_a')}\right)^2 \qquad \text{Eq. 21}$$

Constitutive equation with complex modulus expression is:

$$F=(K'(\omega)+iK''(\omega))\Delta \qquad \text{Eq. 22}$$

By the definitions, the loss factor is $\eta(\omega)=K''(\omega)/K'(\omega)$; the dissipated energy is $Wd(\omega) = \pi K''(\omega)\Delta^2$; and relaxation time is $\tau = C2/K2$, where

$$K'(\omega) = \frac{(K1+K2)(\omega\tau)^2 + \tau K1}{1+(\omega\tau)^2} \qquad K''(\omega) = \frac{C2\omega}{1+(\omega\tau)^2} \qquad \text{Eq. 23}$$

The applied story force (Fs) on the structures for harmonic excitation with the frequency of ω can be expressed by:

$$Fs = K'(\omega)\Delta \sin(\omega t) + \frac{K''(\omega)}{|\omega|}\Delta\omega \cos(\omega t) \qquad \text{Eq. 24}$$

The restore force in Equation10 can be recast in form of complex stiffness,

$$f(x,\dot{x}) = K'(\omega)x + \frac{K''(\omega)}{|\omega|}\dot{x} \qquad \text{Eq. 25}$$

From above the complex stiffness analysis, the toggle brace damper system shows visco-elastic behavior. In other words, the efficiency of the toggle brace damper system largely relies on the brace stiffness, damping value, and the structural stiffness. The comparison of the dynamic response, the SDOF system from SAP2000 analysis, with theoretical solution for the harmonic excitation is shown in Figure 4 with good matching results.

Results for a practical example

Following results are calculated based on the bay length $L = 31'$; story height $H = 12'$-$6''$; lower brace angle $\alpha_a = 19^0$ with the length of $L_a = 24'$; upper brace angle $\alpha_b = 29.5^0$ with the length of $L_b = 9'$-$5''$. The cross-section area of both braces $A_{a,b} = 20\ in^2$. The story stiffness $K1 = 462$ *kips/in* or equivalent structure radian frequency of *1.216rad/s*. For target story drift $\Delta = 0.8$ *inch*, the motion amplification factor $Am = 2.9$ and force amplification factor $Amf = 6.1$. The damping value $c = 20$ *kips-sec/inch* and the excitation radian frequency $\omega = 1.257$*rad/s*. The efficiency of the toggle brace system in terms of the dissipated energy, phase angle, and loss factor with different brace stiffness, damping value, and story stiffness along with varied excitation frequencies are listed from Figures 5 to 17.

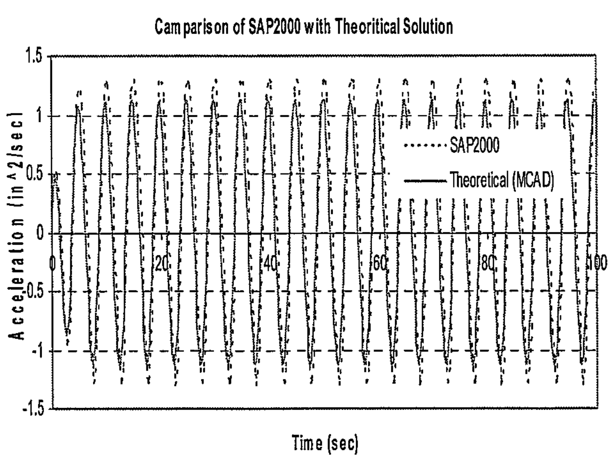

Fig. 4 Harmonic Dynamic Response

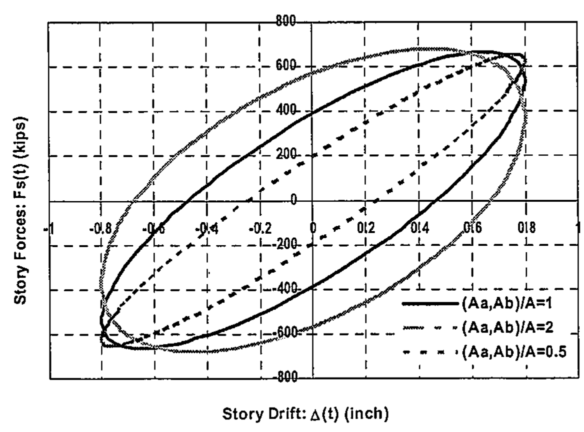

Fig 5 Hysteric Loop for Varied Brace

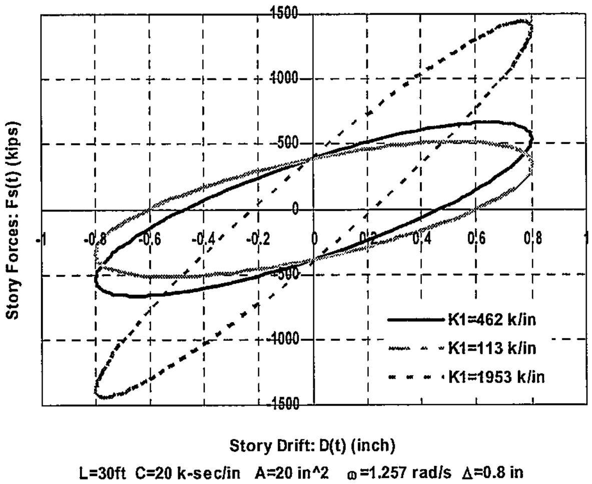

Fig 6 Hysteric Loop of Varied Structural

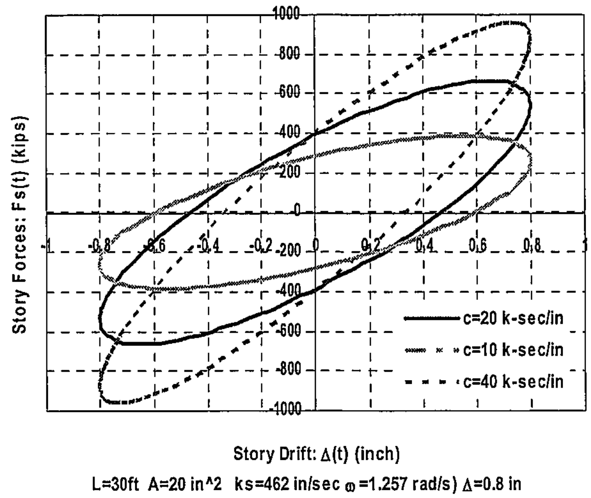

Fig 7 Hysteric Loop of Varied Damping

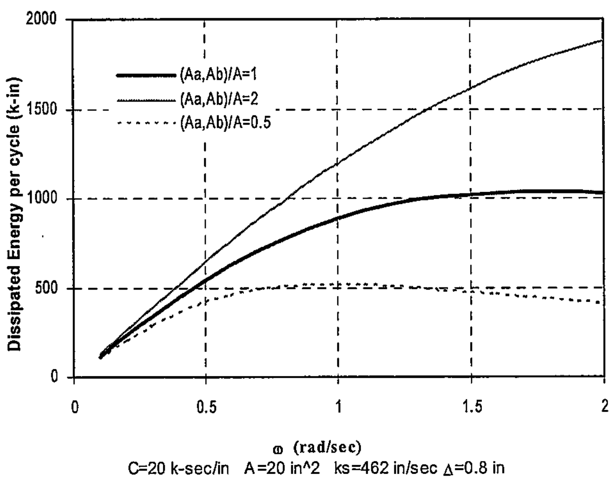

Fig 8 Dissipated Energy of Varied Brace Stiffness

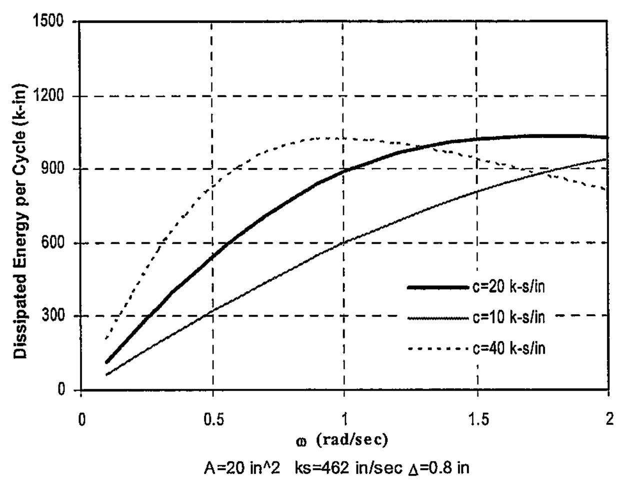

Fig 9 Dissipated Energy of Varied Damping Value

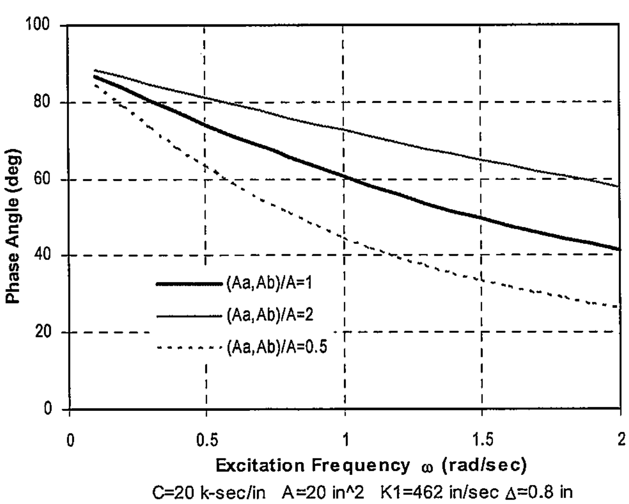

Fig 10 TBD Phase Angle of Varied Brace Stiffness

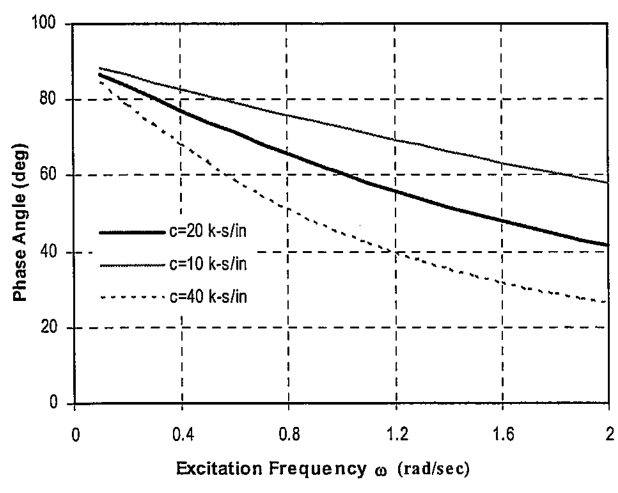

Fig 11 TBD Phase Angle of Varied Damping Value

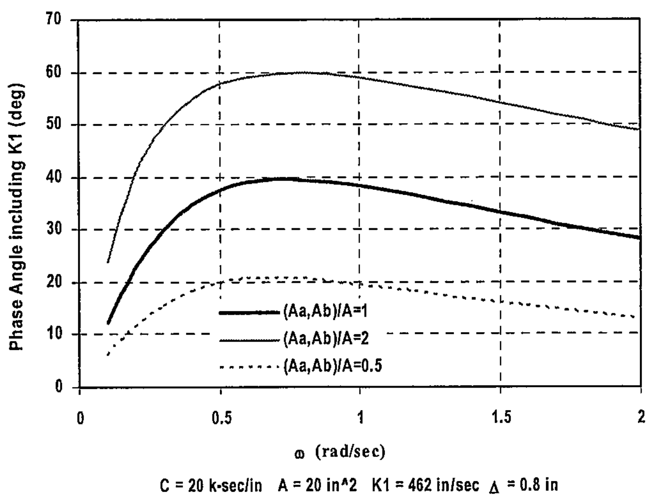

Fig 12 Phase Angle with K1 of Varied Brace Stiffness

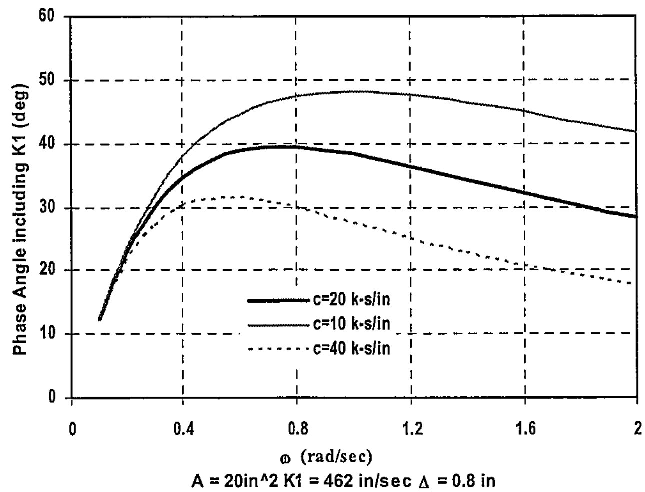

Fig 13 Phase Angle with K1 of Varied Damping Value

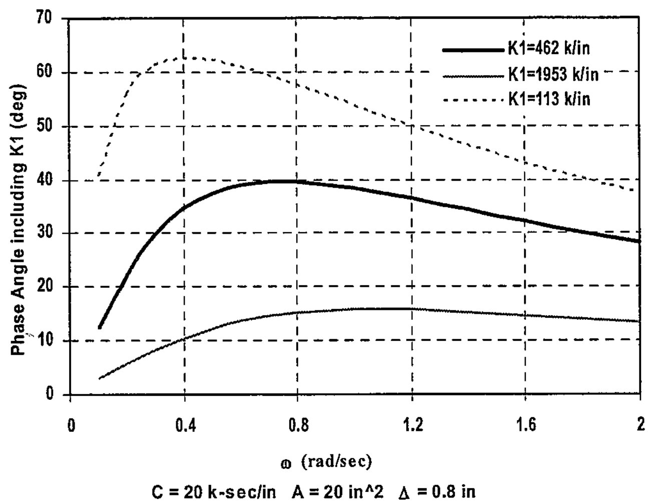

Fig 14 Phase Angle with Varied Structural Stiffness

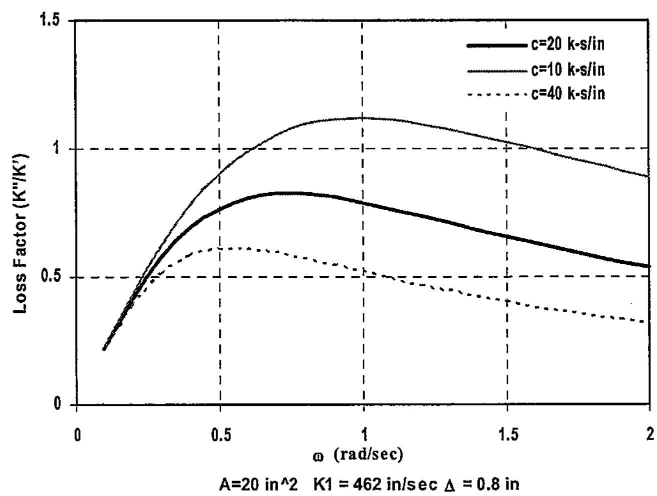

Fig 15 Loss Factor of Varied Damping Value

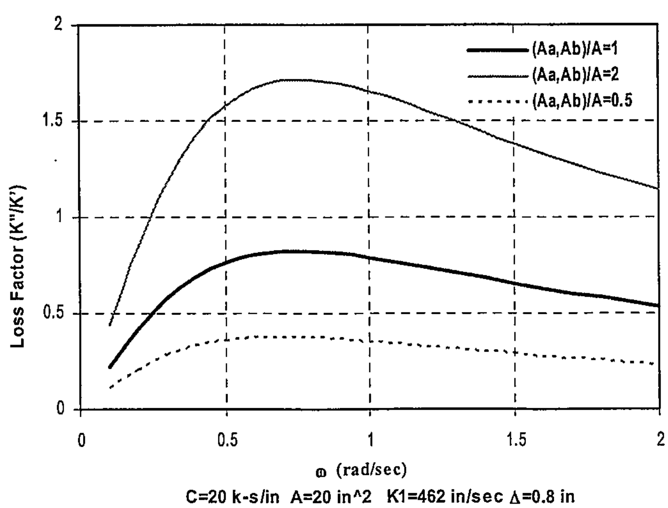

Fig 16 Loss Factor of Varied Brace Stiffness

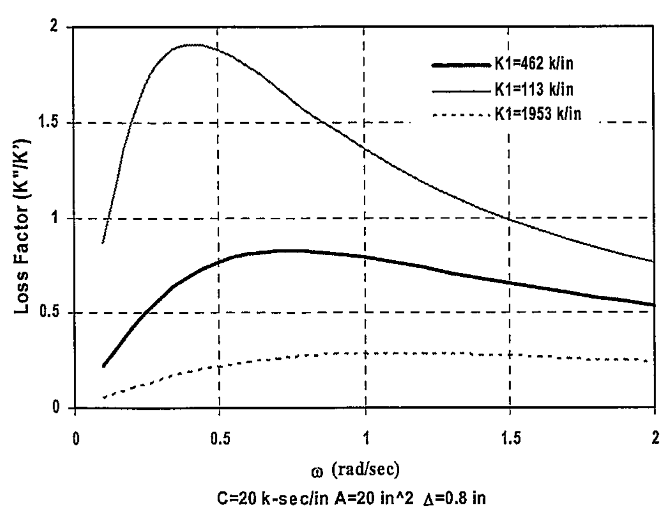

Fig 17 Loss Factor of Varied Structural Stiffness

Effect of Toggle Brace Stiffness on the Efficiency of the Damper System

In the analysis, the area of the toggle brace are varied from $10in^2$, $20in^2$, and $40in^2$ shown in Figure 5 under the fixed excitation frequency. The results show that increasing the stiffness of brace member greatly increases the area of the hysteric loop area or dissipated energy. This implies that adding toggle brace stiffness will improve the efficiency of the toggle brace damper system. The Figure 8 shows that the energy (*Wd*) will be dissipated as the excitation frequency rising and reach the peak, gradually drop as the frequency of the excitation gets high. The stiffer toggle brace will improve the efficiency of the toggle brace system even at the high frequency contents of the excitation.

The phase angle is another important index to reveal the efficiency of the toggle brace system. The phase shift, which is a function to the damper and toggle stiffness, is what causes the reduced effectiveness of the system. When this phase angle approaches zero degrees, the device will be completely ineffective, regardless of the device damping ratio. Two types of the phase angles are discussed to reflect the toggle brace system combined with or without story stiffness. The first phase angle of the toggle brace damper system is defined by $tan^{-1}(K2/\omega C2)$, which represents the toggle braces in series spring model (Maxwell model). The result is shown in Figure 10 that the phase angle will shift more as the excitation frequency getting higher. The higher stiffness of the toggle braces definitely give less of the phase angle shift. The second phase angle is defined by $tan^{-1}(K''(\omega)/K'(\omega))$, which includes structural stiffness to represent the iso-strain model (Voigt model). The results in Figure 12 show that stiffer brace has the larger phase angle, so the damper will be more effective. The phase angle shifting is also a function of the excitation frequency.

The loss factor can be defined by ratio of loss modulus to storage modulus. From Equation 24, storage modulus $K'(\omega)$ provides the "elastic" stiffness of the toggle brace system and loss modulus $K''(\omega)$ represents velocity-dependent or viscous stiffness of the device. The relationships of the loss factor are plotted as the function of the excitation frequencies shown in figure 16. The loss factor gradually decline as the excitation frequency increased at high frequency range. The stiffer braces have larger loss factor. From above discussion, the stiffer of the braces is the more efficient of the toggle brace damper system will be.

Effect of Damping Value on the Efficiency of the Toggle Brace Damper System

Three damping values of 10 kips-sec/in, 20 kips-sec/in, and 40 kips-sec/in are investigated and plotted in Figure 7. Higher damping value will add additional stiffness to the toggle brace damper system but more energy may not be dissipated because it depends on the excitation frequency (see Figure 9 and phase angle shift). Small damping value produces more energy dissipated for the high-frequency range. The phase angle for both, including and not including structural stiffness in Figures 11 and 13, are more favorable towards small damping value of the damper. Similar trends are also shown in figure 15 for the loss factor versus excitation frequency. It implies that the larger damping value of the viscous damper won't increase the dissipated energy.

Effect of Structural Stiffness on the Efficiency of the Toggle Brace Damper System

The toggle brace damper system with different structural stiffness (113 kip-in, 462 kip-in and 1953 kip-in) is also studied. From Figure 6, it can be seen that the area of hysteric loop has little change with the structural stiffness. From the phase angle plot in Figure 14, the toggle brace damper system will be more efficient for the softer structural system. In other words, the stronger of the toggle brace damper system will bring more resistance. Similar results are also obtained from Figure 17.

Conclusions

The efficiency of the toggle brace damper system is directly related to the effective stiffness of the braces. The elongation of the toggle braces must be included in the analysis. The system works like a Maxwell model, where the damper is in series with linear springs. The damper is acting at its full capacity if the braces are very stiff, otherwise the force transmitted to damper can be very small. Adding damping value will not increase efficiency of the damper system. Because of the sensitivity of the system to toggle stiffness, engineers must carefully consider all other sources of the deformation, which may produce loss of the effectiveness in the toggle brace system, such as floor diaphragm deformations; axial and flexural deformation of columns and beams; panel zone deformation; the damper linkage; and all connections associated with. All structural elements in the load path between the damping system and the lateral force resisting system must be included in the analysis to accurately predict the efficiency of the damper system. Also, as noted from complex modulus analysis, the efficiency of the toggle brace damper system also varies with the excitation frequencies.

References

Berton S. and Bolander, E.B., 2005, Amplification System for Supplemental Damping Devices in Seismic Applications, *J. Struct. Engrg, ASCE 131(6),* 979-983.

Costantinou, M.C., Tsopelas P., Hammel W. and A. Sigaher, 2001, Toggle Brace Damper Seismic Energy Dissipation Systems, *J. Struct. Engrg., ASCE 127(2)*, 105-112.

Hanson R.D., and Soong, T.T., 2001, Seismic Design with Supplemental Energy Dissipation Devices, *EERI Monograph Series,* MNO-8, page 47-53.

Huang C., 2004, Parametric Study for Motion Amplification Devices with Viscous Dampers, *13th World Conference on Earthquake Engineering, Paper No. 3060,* Vancouver, Canada, August 1, 2004.

Hwang J.S., Huang Y.N., and Hung Y.H., 2005, Analytical and Experimental Study of Toggle Brace Damper Systems, *J. Struct. Engrg, ASCE 131(7),* 1035-1043.

McNamara, R.J., Huang, C., and Wan V., 2000, Viscous Damper with Motion Amplification Device for High-Rise Building Applications, *Advanced Technology in Structural Engineering, ASCE Structures Congress 2000,* in Philadelphia, PA, May 8-10, 2000.

McNamara, R.J. and Huang, C., 2000, A Efficient Damper System for High-Rise Building, *Fifth Conference on Tall Building in Seismic Regions*, in Los Angles, CA, May 5, 2000.

Ribakov, Y. and Reinhorn, A.M., 2003, Design of Amplified Structural Damping Using Optimal Considerations, *J. Struct. Engrg, ASCE 129(10),* 1422-1427.

Shao, D. and Miyamoto, K., 2002, Seismic Performance Improvement of a Torsional Irregular Concrete Shear Wall Building Using Toggle Brace Dampers, *Proceeding of Seminar on Response Modification Technologies for Performance Based Seismic Design, ATC-17-2,* Los Angles, CA, 2002.