INDIRECT FLEXURAL CRACK CONTROL OF CONCRETE BEAMS AND ONE-WAY SLABS REINFORCED WITH FRP BARS

Carlos E. OSPINA [1] Charles E. BAKIS [2]

[1] Berger/ABAM Engineers Inc., Federal Way, Washington, USA
[2] ESM Dept, The Pennsylvania State University, University Park, Pennsylvania, USA

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1 INTRODUCTION

This paper presents the fundamentals of an indirect flexural crack control procedure for the serviceability design of concrete beams and one-way slabs reinforced with FRP bars. Rather than calculating crack widths, the proposed procedure follows the format of the flexural crack control model for structural concrete design recommended by ACI 318M-05 [1] in which cracks are controlled by specifying a maximum permissible reinforcing bar spacing.

The objective of proposing a flexural crack control procedure in terms of maximum bar spacing rather than a crack width calculation and comparison with allowable limits is to avoid all the impracticalities associated with direct crack width measurements. The proposed procedure does not represent a significant departure from the current flexural crack control model for FRP-reinforced concrete structures reported in ACI 440.1R-06 [2]. Instead, the proposed approach is seen as a re-arrangement of the flexural crack control rules of [2]. One additional advantage of the proposed crack control equation is that it can be used for both FRP- and steel-reinforced concrete design. The proposed procedure explicitly accounts for the dominant effects that bar cover and FRP reinforcement stress level, stiffness and bond properties have on flexural cracking.

2 BACKGROUND

2.1 Flexural Crack Control in Steel-reinforced Concrete Members

In 1999, the ACI 318 code replaced the traditional z-factor approach (based on the Gergely-Lutz [3] model) for crack control of steel-reinforced concrete flexural members with an indirect procedure that calculates a maximum bar spacing. The exposure condition dependence on crack control, previously associated with the z-factor approach, was also eliminated. This serviceability design philosophy change was mainly the result of concerns of ACI Committee 318 about the adequacy of the empirically-tuned Gergely-Lutz model to predict crack widths in flexural members with large bar covers and research results showing no conclusive evidence linking reinforcement corrosion with crack widths.

The new crack control procedure in [2] is based on the work of Frosch [4], who, based on fundamental crack control concepts introduced originally by Broms [5], developed the following equation to calculate the maximum crack width at the tension face of a reinforced concrete flexural member:

$$w = 2\frac{f_r}{E_r}\beta\, k_b\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \tag{1}$$

In Eq. (1), f_r is the reinforcing bar stress, calculated assuming elastic-cracked conditions, E_r is the modulus of elasticity of the reinforcement, β is the ratio of the distance from the neutral axis to the tension face of the member to the distance from the neutral axis to the centroid of the tensile reinforcement, d_c is the cover thickness from the tension face to the center of the closest reinforcing bar, s is the bar spacing (taken as the member width for the case of a single bar) and k_b is a bond coefficient that accounts for the bond characteristics of the reinforcement.

Although Frosch's equation was originally envisaged for steel-reinforced members, it is expressed in Eq. (1) in a more general form so that it can be applied regardless of whether the reinforcement is steel or FRP.

Acknowledging that crack spacing and crack width are functions of the bar spacing, Frosch [4] proposed that crack control could also be achieved by limiting the reinforcement spacing based on acceptable crack width limits. This rationale led to a rearrangement of Eq. (1) to solve for the maximum permissible bar spacing as

$$s = 2\sqrt{\left(\frac{w\,E_r}{2f_r\beta\,k_b}\right)^2 - d_c^2} \tag{2}$$

Assuming cracked-elastic conditions, β is calculated as

$$\beta = 1 + \frac{d_c}{d(1-k)} \tag{3}$$

with

$$k = \sqrt{2\rho_r n_r + (\rho_r n_r)^2} - \rho_r n_r \tag{4}$$

where n_r is the modular ratio and ρ_r is the reinforcement ratio. Taking into account that β varies with the cover, Frosch [4] developed the following equation to simplify the bar spacing calculation:

$$\beta = 1 + 0.0031 d_c \qquad (d_c \text{ in mm}) \tag{5}$$

Frosch also developed a simplified equation which provided a discontinuous representation of Eq. (2). Details can be found in [4]. ACI Committee 318 made some modifications to this equation and adopted a new crack control equation to evaluate the maximum bar spacing for the 1999 code. The equation is as follows:

$$s = 380\left(\frac{280}{f_s}\right) - 2.5\,c_c \le 300\left(\frac{280}{f_s}\right) \qquad (f_s \text{ in MPa}) \tag{6}$$

where f_s is the steel reinforcement stress at service level and c_c is the clear cover, i.e. d_c minus half the bar diameter of the bottom layer of bars.

Equation (6) is simply an alternate (and simpler) representation of Eq. (2). Crack control through either Eq. (2) or Eq. (6) is said to be "indirect" because the maximum bar spacing is indirectly constrained by a limiting crack width, w. Notice that Eq. (6) uses the subscript "s" to refer to steel reinforcement. In those cases where there is only one bar nearest to the extreme tension face, the maximum bar spacing is defined as the width of the extreme tension face. In members with multiple layers of tension reinforcement, Eq. (6) is defined based on the assumption that only the bottom layer affects crack widths.

Although ACI 318M-05 does not make any explicit reference to a limiting crack width value associated with Eq. (6), Frosch clearly showed that the precursor equation of Eq. (6) was indirectly tied to a controlling crack width range that varies between 0.4 mm and about 0.52 mm. The former is the allowable crack width for interior exposure conditions in earlier ACI 318 code versions. The latter represents a 30% variation due to the inherent scatter associated with cracking.

In lieu of more detailed calculations, [1] allows f_s to be taken as $0.67f_y$. This stress level slightly exceeds the 0.6 f_y level assumed in earlier ACI 318 code versions. The increased stress level is the result of the new load factors introduced in the ACI 318 code in 2002. As a matter of fact, Eq. (6) differs slightly from the ACI 318-99 crack control equation to reflect the load factor change introduced in the 2002 code. Notice that if one were to interpret the ACI 318 code provision as being dependent on a limiting crack width, the higher stress level should lead, in turn, to a higher controlling crack width lower bound equal to (0.4 mm)(0.67/0.6) = 0.44 mm and, similarly, to an upper bound of 0.58 mm. This clarification should, in principle, be added to the wording of the ACI 318M-05 crack control design provision.

For the case of steel reinforcement, Fig. 1 shows the recommended maximum bar spacing according to both [1] and Eq. (2) in terms of the concrete cover, d_c. The bar spacing predictions have been calculated assuming $f_r = 0.67f_y$ and k_b =1.0. Maximum specified bar spacing values are calculated based on limiting crack width values of 0.44 mm and 0.58 mm. In the bar spacing predictions according to [1], the clear cover was defined as d_c minus half the diameter of a No. 7 (19.1 mm diameter) bar, and it was assumed that all the bars are of the same size.

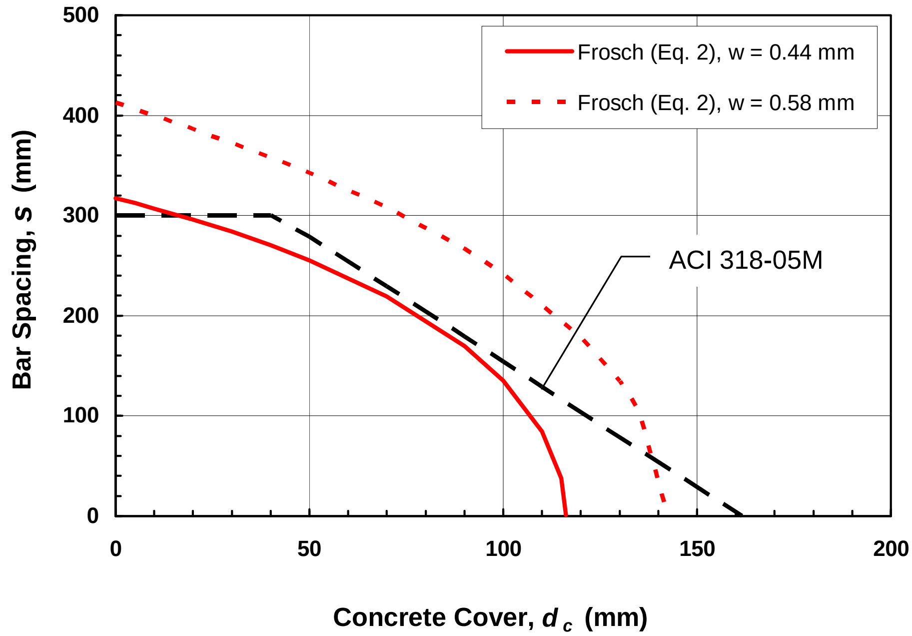

Fig. 1 ACI 318M-05 Flexural Crack Control Provisions for Steel Reinforcement.

Figure 1 confirms Frosch's observation that the maximum bar spacing requirement is indirectly associated with a limiting crack width range. Equation (6) predictions reasonably match the maximum bar spacing predictions per Eq. (2) for w = 0.44 mm except for concrete covers that exceed about 110 mm. Beyond this cover level, the bar spacing predictions are grossly on the liberal side. It is worth noting, however, that covers this large are rarely found in typical beam and slab construction. One alternative for improving the design recommendation would be to adopt a tri-linear curve by prescribing a maximum concrete cover. The maximum cover will depend on the flexural member type and target durability requirements.

2.2 Flexural Crack Control in FRP-reinforced Concrete Members

Prior to 2006, crack widths for FRP-reinforced concrete design in ACI 440.1R [2] were predicted based on a modified version of the original Gergely-Lutz model. The modification was imposed to account for the observed range of moduli and bond properties of various types of FRP reinforcement.

The crack control provisions in [2] are also based on the work of Frosch. However, unlike [1], the crack control procedure is "direct" in the sense that crack widths are calculated and then compared directly with maximum allowable crack width limits depending on the exposure condition. The crack width limits in [2] are 0.5 mm and 0.7 mm, for exterior and interior exposure conditions, respectively. For FRP reinforcing bars with bond properties similar to those of ordinary steel bars, k_b is assumed equal to one. For FRP reinforcing bars with better bond characteristics, k_b is taken less than unity. For FRP reinforcing bars with inferior bond characteristics, k_b is assumed greater than unity. If the k_b value is unknown, it shall be taken as 1.4 for non-smooth bars.

The reference crack width limits of 0.5 and 0.7 mm were adopted from the 2000 Canadian Highway Bridge Design Code [6]. These crack width limits are more relaxed than those associated with conventional reinforced concrete design due to the superior corrosion resistance of FRP. The k_b values are based on the work by Bakis et al [7]. These values reflect the fact that the calibration of the bond coefficient using Frosch's model in light of experimental evidence from tests on beams and one-way slabs with FRP reinforcing bars [8-15] renders k_b values that are about 19% greater than those resulting from using the modified Gergely-Lutz equation based on the same data.

3 PROPOSED FLEXURAL CRACK CONTROL MODEL

3.1 Validation of ACI 440.1R-06 Flexural Crack Control Model

To the authors' best knowledge, other than the exercise conducted in [7] to re-calibrate the k_b values to be used in conjunction with the crack control equation of [2], the crack width predictions using Eq. (1) have not been compared directly with observed crack width measurements taken from FRP-reinforced beam or one-way slab tests. This comparison is paramount to ensure that any simplification or reorganization of the current ACI 440.1R-06 crack control equation, as intended through the proposed crack control model that is presented later in this paper, will have an adequate level of safety.

Figure 2 shows a comparison between maximum crack width predictions per Eq. (1) and observed maximum crack widths from test results reported in the literature [8-15]. Crack widths included in Fig. 2 correspond to only one load level for each of the beam test results available. In [9, 12 and 15], reported maximum crack width values correspond to 90% percentile crack widths, i.e. mean crack width plus 1.28 times the standard deviation. In [10, 11, 13 and 14], the maximum crack width stands for that in the widest crack across the flexural zone of the test specimen. The vast majority of test specimens were reinforced with GFRP bars whereas some had CFRP reinforcing bars and only a few had AFRP reinforcing bars, all with various types of surface treatments. Smooth bars and grids are not included in the body of data.

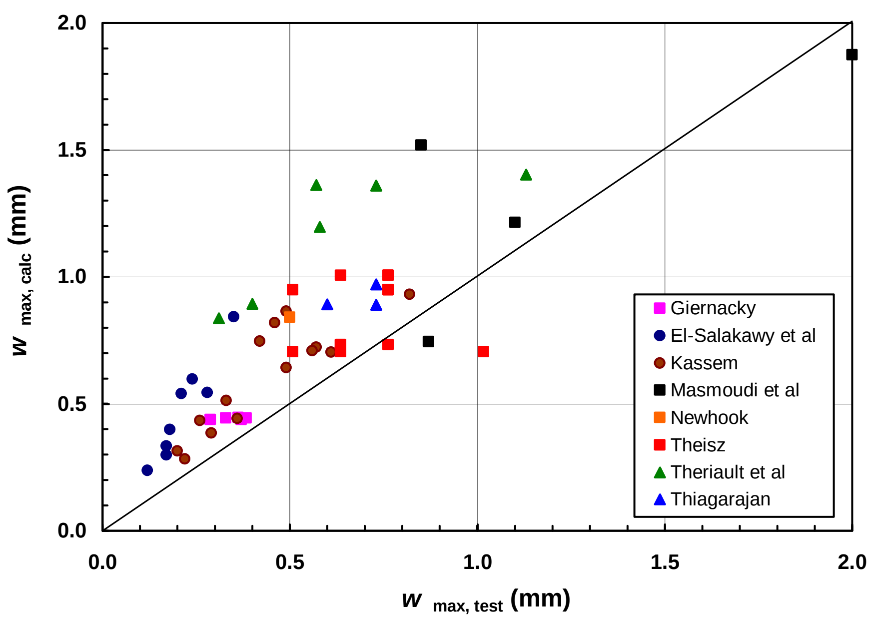

Fig. 2 Comparison Between Eq. (1) Predictions and Experimental Evidence.

Selected observed crack width data points are associated with a range of FRP strain values considered to be typical of service load levels in conventional field applications. In the test data from [11, 13 and 14], the selected FRP strain levels correspond to those at about 50% of the nominal bending capacity of the beams, which translate into FRP strains varying from 4500 to 9000 $\mu\varepsilon$ for both GFRP and CFRP. In the remainder of the references, the FRP strain levels varied from about 1100 $\mu\varepsilon$ to 5700 $\mu\varepsilon$ for CFRP and from 1300 $\mu\varepsilon$ to 4300 $\mu\varepsilon$ for GFRP. For the beams with AFRP, the strain varied from about 3400 to 4300 $\mu\varepsilon$. The FRP strain values used in the maximum crack width calculations were obtained either directly from strain gages mounted on the FRP bars or through calculations assuming cracked-elastic conditions. A value of k_b = 1.4 was assumed in the maximum crack width calculations. Figure 2 shows that the ACI 440.1R-06 equation, based on the model proposed by Frosch, leads to conservative maximum crack width predictions for the majority of concrete beam tests examined.

3.2 General Model for Indirect Flexural Crack Control of Concrete Beams and One-way Slabs

The starting point for the development of the proposed indirect flexural crack control equation is Eq. (6). To be able to control flexural cracks in either steel- or FRP-reinforced beams or one-way slabs, it is necessary to introduce the effect of the elastic modulus and bond characteristics of the reinforcement. In order to control flexural cracks in structures subject to very aggressive exposure conditions or, on the contrary, to allow wider flexural cracks in concrete beams and one-way slabs reinforced with non-corroding reinforcement, such as FRP, the following model is proposed.

Normalizing the first left hand side term of Eq. (6) by the crack width and elastic modulus ratios, using 0.44 mm and 200,000 MPa as reference values, and introducing the bond effect through the k_b coefficient, leads to the following new equation:

$$s = 380\left(\frac{280}{f_r}\right)\left(\frac{E_r}{200{,}000}\right)\left(\frac{w}{0.44}\right)\frac{1}{k_b} - 2.5\,c_c \le 300\left(\frac{280}{f_r}\right)\left(\frac{E_r}{200{,}000}\right)\left(\frac{w}{0.44}\right)\frac{1}{k_b} \qquad (7)$$

with f_r and E_r in MPa.

In Eq. (7), the k_b values can be defined as in [2] because the proposed equation does not represent a significant departure from Eq. (2). Equation (7), in turn, leads to

$$s = 1.2\frac{E_r w}{f_r k_b} - 2.5\,c_c \le 0.95\frac{E_r w}{f_r k_b} \qquad (8)$$

which can be re-written, in terms of the reinforcement strain, as

$$s = 1.2\frac{w}{\varepsilon_r k_b} - 2.5\,c_c \le 0.95\frac{w}{\varepsilon_r k_b} \qquad (9)$$

In theory, every term in Eq. (6) should have been normalized. Nevertheless, the slope of the ACI 318M-05 design curve (see Fig. 1) was kept as -2.5 for the proposed model. Further evaluation (not reported herein) of Eq. 2 shows that the ellipsoidal-shaped design curve that defines the maximum bar spacing requirements as a function of the concrete cover changes only in size and not in aspect ratio when the input parameters, i.e. limiting crack width, FRP elastic modulus and bond coefficient, are varied. As a result, a constant slope for the simplified design equation was deemed appropriate. If the slope were normalized by the elastic modulus and crack width ratios, the resulting simplified design curve may drastically diverge relative to Eq. (2) predictions. This indicates that Eq. (6), despite being simplistic, is not fully consistent with the intent of the cracking model proposed by Frosch.

Equation (8) is of a general nature. Hence, it can be used as a flexural crack control equation for structural concrete design provided the concrete member is reinforced with bars whose response is described by Hooke's law at the stress level at which the flexural cracks are to be controlled. This means that the proposed flexural crack control model could be used for the serviceability design of concrete members reinforced with either black steel bars, epoxy-coated bars or FRP bars, provided that the bond characteristics of the reinforcement are duly represented through the k_b factor. In addition to the prominent effect of the concrete cover and the effects of the stress level, the elastic modulus and the bond characteristics of the reinforcement, Eq. (8) shows explicitly the influence of the limiting crack width on the prescribed reinforcing bar spacing. The bar spacing dependence on crack width is advantageous because it gives the designer latitude to determine the level of flexural cracking to be controlled. Allowable crack width limits for steel-reinforced concrete structures are reported in [16] and [17] for a wide variety of exposure conditions. In case the reinforcement spacing is constrained to a given value, flexural cracking can also be controlled by prescribing the stress level in the reinforcement.

3.3 Proposed Model for Indirect Flexural Crack Control of Concrete Beams and One-way Slabs with FRP Reinforcement

The proposed crack control equation for FRP-reinforced beams and one-way slabs is a special case of Eq. (8). For convenience, interior exposure conditions are assumed. Substituting w = 0.7 mm

(0.028 in), which is the value adopted by ACI 440.1R-06 for interior exposure conditions, into Eq. (8), and simplifying, results in

$$s = 0.8\frac{E_r}{f_r k_b} - 2.5\,c_c \le 0.7\frac{E_r}{f_r k_b} \qquad (f_r \text{ and } E_r \text{ in MPa}) \qquad (10)$$

Figure 3 shows maximum bar spacing predictions from Eqs. (2) and (10) as a function of the concrete cover d_c for a member with GFRP reinforcing bars, assuming E_r = 40 GPa, f_r = 80 MPa, and k_b = 1.4, for limiting crack widths of 0.7 and 0.91 mm in the case of Eq. (2). The latter represents a 30% variation from the lower crack width limit.

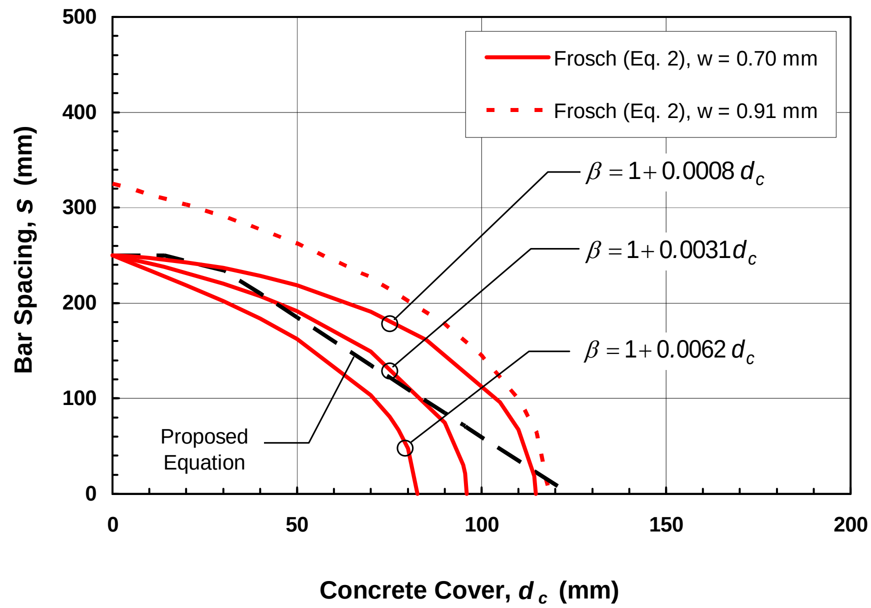

Fig. 3 Proposed Flexural Crack Control Provisions for GFRP-reinforced Concrete Members

The maximum bar spacing predictions per Eq. (2) associated with a limiting crack width of 0.7 mm shown in Fig. 3 are based on the following β relationships:

- $\beta = 1 + 0.0062\,d_c$, which represents conditions typical of shallow members, d = 230 mm
- $\beta = 1 + 0.0008\,d_c$, which represents deeper members, d = 1800 mm
- $\beta = 1 + 0.0031 d_c$, which is the value assumed by Frosch for steel-reinforced concrete design.

The maximum bar spacing predictions for the 0.91 mm crack width limit are based on $\beta = 1 + 0.0031 d_c$ only.

The different β equations are intended to account in a simplified manner for the effect that flexural depth, reinforcement ratio, and concrete cover may have on β. These variables may vary significantly, especially in beam construction. One of the underlying assumptions behind each β definition is the assumption k = 0.3. This value may be considered typical for steel-reinforced beam construction. While it is understood that a member with FRP reinforcement ratio may well have a lower k compared to a comparable steel-reinforced concrete member with similar reinforcement ratio, members with FRP reinforcement will generally have higher reinforcement ratios to avoid serviceability problems. Their higher reinforcement ratio will lead to a deeper neutral axis in the cracked-elastic stage. It is for this reason that k = 0.3 may not be an unrealistic assumption for FRP-reinforced member design.

Figure 3 shows that, for a given concrete cover, the maximum bar spacing requirement is less restrictive for deeper members. The figure also shows that Frosch's assumption for β seems reasonable as an average value for serviceability design. It is worth noting that the proposed crack control procedure has already built-in the β definition proposed by Frosch. It is possible to verify the effect of the flexural depth, reinforcement ratio and concrete cover on the maximum bar spacing calculations by simply invoking Eq. 2. The figure also shows that Eq. (10) provides a reasonable representation of Frosch's model for crack control of concrete members with GFRP reinforcing bars for the given crack width limits considered.

Figure 4 shows the effect of bond quality of FRP reinforcement on the maximum bar spacing requirements. The assumptions are the same as those used in Fig. 3 except that two k_b values, 1.0 and 1.4, respectively, have been used in Eq. (2) and Eq. (10) bar spacing predictions. A k_b value of 1.0 represents FRP reinforcement with bond characteristics similar to those of ordinary steel bars. A k_b value of 1.4 represents FRP reinforcement of inferior bond characteristics. The bar spacing has been calculated assuming a limiting crack width of 0.7 mm and $\beta = 1 + 0.0031 d_c$.

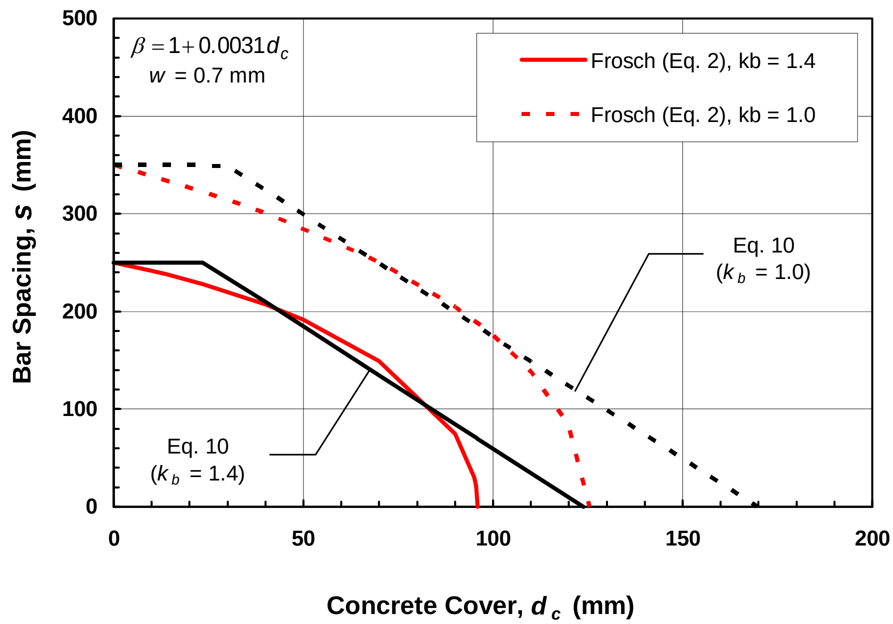

Fig. 4 Effect of Bond Quality of FRP Reinforcement on Flexural Crack Control Requirements.

Figure 4 shows the detrimental effect of inferior bond on flexural crack control requirements. For a given concrete cover, the required FRP bar spacing necessary to comply with the target crack width limit decreases as the quality of bond of FRP reinforcement decreases, i.e. as k_b increases. The figure also shows that Eq. (10) adequately represents Frosch's equation bar spacing predictions for both k_b values examined for practical concrete cover values.

4 EVALUATION

This section examines the validity of the proposed indirect crack control procedure in light of the same experimental evidence used to validate Frosch's crack control model. The examination will be made by comparing predicted with observed crack widths. This means it is necessary to rewrite Eq. (8) and solve for the maximum crack width. This leads to

$$w = \frac{f_r k_b}{1.2 E_r}\left(s + 2.5 c_c\right) \geq \frac{s f_r k_b}{0.95 E_r} \qquad (11)$$

Figure 5 shows the crack width predictions per Eq. (11) and those of ACI 440.1R-06 in light of observed crack widths from the databank assembled by Bakis et al. [7]. The underlying assumptions for the calculation of crack widths based on observed FRP strains are similar to those drawn for plotting Fig. 2. Filled symbols show the maximum crack width predictions per [2] whereas empty symbols show maximum crack predictions per Eq. (11).

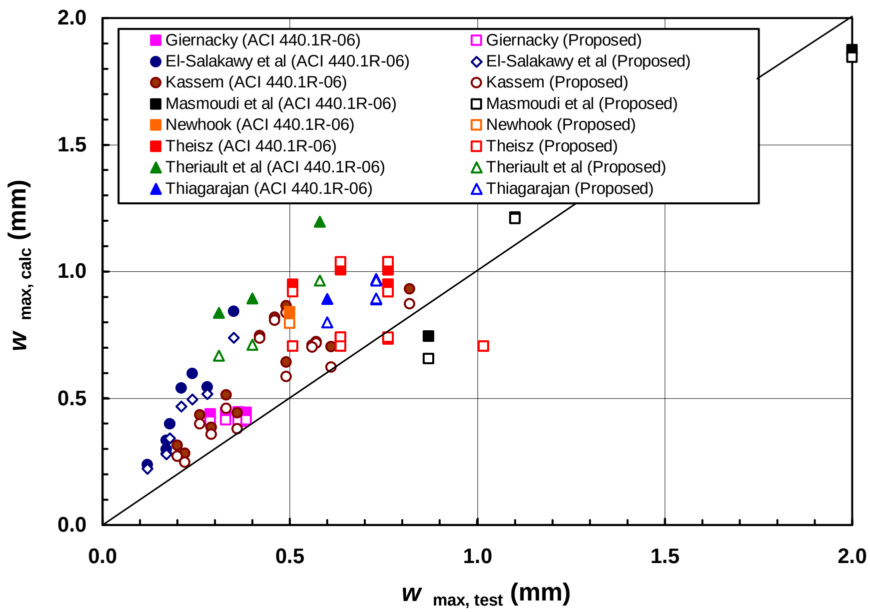

Fig. 5 Comparison Between ACI440.1R-06 and Eq. (11) with Experimental Evidence.

Figure 5 shows that crack width predictions per Eq. (11) are similar to those of ACI 440.1R-06. The predictions are also conservative except for a few test results. The former observation is not surprising because Eq. (11) has been shown above to follow the general trends predicted by the ACI 440.1R-06 crack control equation. The discontinuous design curve defined by Eq. (11) leads in this case to predicted maximum crack widths that are close to those observed in the tests. In all cases, predicted crack width values were always lower than the upper limit defined in the right hand side of the inequality.

The main advantage of the proposed general equation (Eq. 8) is that it can be applied to both steel- and FRP-reinforced concrete beams and one-way slabs, as long as the elastic stiffness and bond characteristics of the reinforcement are properly described.

Selection of the limiting crack width or crack width range is closely linked with the intended use of the structure. The model allows control of different levels of cracking, including those associated with structures in aggressive environments or where water tightness is to be enforced, and, conversely, in situations where wider cracks may be acceptable due to the use of corrosion-resistant reinforcement such as FRP. Notice that the crack control equation of ACI 318M-05 may not be readily used in situations in which crack widths either much smaller than 0.4 mm or wider than 0.52 mm are to be controlled because the bar spacing equation is only valid for a crack width range that falls within these limits. Crack control requirements in structures where water tightness is needed are a perfect example of cases where the 0.4 mm crack width limit may be inappropriate.

Equation (8) also gives latitude to designers to select a target FRP strain to ensure compliance with both the serviceability and ultimate limit states. The goal in this case is to select the proper reinforcement strain level so as to avoid excessive deflections or brittle failure, particularly when GFRP reinforcement is used and sustained loads are present.

5 CONCLUSIONS

This paper presents an indirect procedure to control flexural cracks in structural concrete beams and one-way slabs. The crack control is exerted by prescribing a maximum bar spacing instead of a maximum crack width. The procedure is said to be indirect because the maximum bar spacing requirement is indirectly linked to a target crack width value that is to be complied with. The proposed procedure can be applied to either steel- or FRP-reinforced concrete beams or one-way slabs.

The proposed crack control model is not considered to be a drastic departure from the ACI 440.1R-06 flexural crack control equation. Instead, it is merely a discontinuous representation of the ACI 440.1R-06 equation re-arranged in terms of the FRP reinforcement spacing. This re-arrangement was implemented to attempt eliminating the impracticalities associated with direct measurement of crack widths in the field amidst the enormous variability of the concrete cracking phenomenon and also to make the ACI 440.1R flexural crack control design provisions consistent in format with the flexural crack control recommendations of the ACI 318 code.

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