SLACK ROPE ANALYSIS FOR MOVING CRANE SYSTEM

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SUMMARY

The reliability and security of rope design for moving crane are very important especially for the nuclear plant in high seismic zones. The conventional linear analysis indicates that a slack rope occurs very likely for severe earthquake load excitation. In other words, the rope will overcome its lifted weight and will go into compression. The nonlinear time history method according to NOG-4154 shall be applied for slack rope design. In order to perform nonlinear time history analysis subject to earthquake excitation, the tension-only nonlinear properties of element shall be taken into account. The designated program – GTStrudl or other nonlinear – program may have such a capability for solving nonlinear dynamic systems. However, the result shows that the current tension-only nonlinear finite element in GTStrudl has the reasonable accurate results with comparing theoretical results for damped single degree-of-freedom (SDOF), but it fails to converge for a large-scale DOF of computer model for trolley-bridge system due to severe nonlinearity of rope. Simplified analysis shall be employed in rope slack nonlinear study. Because only vertical mass of lifted weight is included, the rope forces caused by horizontal earthquake load in high modes are very small and can be neglected. According to this dynamic characteristic, we simplify and use two-degree-of-freedom (2DOF) structural systems to represent a multi-DOF of bridge-trolley with lifted load system in vertical direction. The results show that this simplification proved to be very accurate and successful. This paper presents a very simple 2DOF nonlinear dynamic model and compares rope forces between linear dynamic analysis and nonlinear slack rope analysis. The results also show that rope force could be much larger than those from conventional linear dynamical analysis varied with rope length. The proposed slip-slack model shows that the brake slip device can limit rope force, predicts the displacement for prescribed design level, and prevents rope failure due to slack rope impact.

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INTRODUCTION

The safety and security of the rope design for moving crane are crucial, especially, for the nuclear industry in high seismic zones. The rope shall be designed and constructed to remain in place and support a critical load during and after a seismic event and has single-failure-proof features such that any credible failure of a single component will not result in the loss of capability to stop and/or hold the critical load. A critical load is defined as any lifted load whose uncontrolled movement or release could result in potential unacceptable off-site radiation exposure. During a severe earthquake, the lifted object may overcome its self-weight and the rope slack. The restoring forces of the rope will be changed from the spring forces to gravity forces while slack rope happens. Attaway [1] discussed the dynamic impact load on the climbing rope for the rope safety design in his memoriam paper for the loss life of his friend, a rock-climber, due to rope failure. He used energy balance principle to calculate dynamic factor due to slack impact load. For the rope system design of the crane system, the slack rope occurs very likely for strong earthquake load. The conventional linear analysis, which modeled the rope as spring element without slack rope, obviously results in an unconservative solution, particularly in severe vertical seismic excitation. Thus, according to seismic safety guideline for crane of ASME NOG-1-2002 [2] and ASCE 4-98 [3], nonlinear time history analysis shall be employed. In order to perform nonlinear time history analysis subject to earthquake excitation, the tension-only nonlinear properties of element shall be taken into account. The designated program – GTStrudl [4] or other nonlinear programs – may have such a capability for solving nonlinear dynamic systems. From nonlinear time analysis, GTStrudl shows the reasonable accurate results with comparing theoretical results for damped Single degree-of-freedom (SDOF), and experiences difficulty to converge for a large-scale DOF of computer model for trolley-bridge system. Simplified analysis for rope system design is apparently very attractive and tentative. Since the rope can only take tension load, in other words, only vertical mass of lifted weight is included, the rope forces caused by horizontal earthquake load can be neglected. According to this dynamic characteristic, a two-degree-of-freedom (2DOF) nonlinear structural system was proposed to represent a multi-DOF of bridge-trolley with lifted load system along the vertical direction for the rope system design. The results show that this simplification proved to be very accurate and successful for slack rope analysis. This paper discusses the slack rope mechanism by using a simple 2DOF nonlinear dynamic model and also compares rope forces between linear dynamic analysis and nonlinear slack rope analysis. The results show that slack rope can produce a larger impact load on the rope system. Conventional linear dynamic analysis of the rope system will significantly underestimate dynamic impact load if the rope slack. To mitigate impact load on the rope due to slack rope, a slip brake device is introduced to limit the rope forces. A mathematical model is proposed for the slip brake device in order to predict the displacement of the lifted object for prescribed design level.
THE MECHANISM OF SLACK ROPE

Fundamental Equation
Typical moving crane system consists of bridge girders with end-tie beam, trolley truck, and the rope system with hang hook for lifting object. A lifted object does not usually participate in any horizontal motion because the rope only takes tension forces. Therefore, for the rope system design, only vertical motion of the crane system is considered. Furthermore, the crane system with lifted load along vertical direction can be simplified as two-degree-of-freedom (2DOF) dynamic system, which is the bridge girders with trolley girt and lifted object. This simplification diagram of an analytical model and definitions of the mass stiffness are shown in Figure 1. Because only the vertical mode of the crane system is considered for rope study, activated mass of the crane bridge for vertical mode is included. The equivalent stiffness of the bridge girders with trolley load girt can be estimated by deflection of girders at the lifted object position under static gravity load of lifted load \( k = P / \Delta \). Activated crane masses can be either approximated by vertical mode of dynamic analysis.

![Simplification model for 2DOF of crane system](image)

The displacement of the crane \( x_1 \) and lifted weight \( x_2 \) have their coordinates started from their static equilibrium position. The fundamental equilibrium dynamic equations of crane system with lifted load can be expressed in 2DOF dynamic system,

\[
m_1 \ddot{x}_1 + [C_1(\dot{x}_1, x_1) - C_2(x_1, x_2, \dot{x}_1, \dot{x}_2)] + [F_1(x_1) - F_2(x_1, x_2)] = -m_1 \ddot{x}_g
\]
\[
m_2 \ddot{x}_2 + C_2(x_1, x_2, \dot{x}_1, \dot{x}_2) + F_2(x_1, x_2) = -m_2 \ddot{x}_g
\]

where the masses of the crane system \( m_1 \) and lifted load \( m_2 \) are simply obtained by their weights divided by gravity acceleration \( g \). The right sides of equations are the ground acceleration input. The damping forces for crane \( C_1 \) and lifted load of rope \( C_2 \) can be defined as the function of the displacements and velocities as follows

\[
C_1(\dot{x}_1) = 2 \xi \omega_1 \dot{x}_1
\]
\[
C_2(x_1, x_2, \dot{x}_1, \dot{x}_2) = \begin{cases} 
2 \xi_2 \omega_2 (\dot{x}_2 - \dot{x}_1) & \text{if } (x_2 - x_1) \geq -xs \\
0 & \text{Otherwise} 
\end{cases}
\]
where $\zeta_1$ and $\zeta_2$ are the critical damping coefficient of the crane system and lifted rope, respectively. If the relative displacement $(x_2-x_1)$ between crane and lifted object overcomes the stretch of the rope $(xs)$, the rope damping forces will be zero. The restoring forces for crane system ($F_1$) and lifted load of rope ($F_2$) can be describe as the function of their relative displacement as shown in Equations 5 and 6.

$$F_1(x_1) = \omega_1^2 x_1$$
Eq.5

$$F_2(x_1,x_2) = \begin{cases} \omega_2^2(x_2 - x_1) & \text{if } (x_2 - x_1) \geq -xs \\ -g & \text{Otherwise} \end{cases}$$
Eq.6

When the lifted object is moving upwards, the relative displacement $(x_2-x_1)$ is less than the stretch of the rope $(xs)$, the rope starts to slack. The restoring force from rope spring force becomes constant downwards gravity force. In other words, the lifted object starts tossing upwards until it reaches the highest point, then drops by gravity forces and begins to stretch the rope again, where it may produce impact load on the rope. The rope stretch $(xs)$ can be calculated by $xs = w2 / k2 = g / \omega_2^2$ where the lifted load reaches its static equilibrium position. The radian frequencies of the crane and lifted load are defined according to their definitions,

$$\omega_1 = \sqrt{k1 \cdot g / w1}$$

$$\omega_2 = \sqrt{k2 \cdot g / w2}$$
Eq.7

and stiffness and weights of the crane system ($w1$, $w2$, $k1$ and $k2$) are defined in Figure 1. Therefore, the dynamic system becomes nonlinear dynamic system for slack rope from Equations 4 and 6.

### Rope Dynamic Load with Initial Pull-Down Displacement

In order to reveal dynamic impact load for the slack rope, assume the crane has infinite rigid stiffness compared to the rope stiffness, the dynamic system is degraded to an SDOF as shown Figure 2. Pull down the lifted object, then release. Assume that the lifted weight has 256 kips, rope length ($L$) is 71 feet, the rope cross section area ($A$) has 8.61 in$^2$, and the rope elastic modulus ($E$) is 14,000 ksi. The dynamic SDOF system with initial condition can be established accordingly and solved.

For the first approach, according to energy balance for the conservative system without energy dissipation, the strain energy at the pull-down position shall be equal to potential energy at the highest point position,

$$K (x_0 + xs)^2 / 2 = W (x_{max} + x_0)$$
Eq.8

where the $x_0$ is the initial pull-down displacement (3 inches). The rope stiffness ($K$) is equal to $EA/L$. and the stretch of the rope $(xs)$ under static gravity load $(W)$ is 1.81 inches by $W/K$. The maximum displacement $(x_{max})$ of lifted object can be reached as high as 3.39 inches by solving Equation 8.
For the second approach, due to nonlinear dynamic system for slack rope, nonlinear time history analysis is necessary and the Runge-Kutta (RK) numerical direct integration method is most widely used for a few DOF dynamic systems. The undamped dynamic response of the rope force and displacement with initial pull down is calculated by the RK method and displayed as function of time, see Figure 3. The same maximum displacements ($x_{max}$) are found in diagram as predicted by energy balance method. The rope force in Figure 3 does not include self-weight of the lifted load.

**Figure 3 - Dynamic Responses due to Initial Pull Down**

For the third approach, solving analytic equation for projectile motion, the theoretical motion solution for undamped dynamic response of the displacement with initial pull down is $x(t) = x_0 \cos(2\pi f t)$. The frequency of the system ($f = 2.325 \text{ Hz}$) can be calculated from Equation 7, where ($f = \omega/2\pi$). Actually, when the lifted object overcomes the stretch of the rope ($x_s$), the rope slacks off. The lifted object will continue to move up with initial velocity, and the required object climbing time ($t_1$) at the time when the rope starts to slack can be calculated by

$$t_1 = \frac{\arccos(-x_s/x_0)}{2\pi f} \quad \text{Eq. 9}$$

and the initial velocity ($v_1$) at this point will be

$$v_1 = 2\pi f x_0 \sin[\arccos(-x_s/x_0)] \quad \text{Eq. 10}$$

The object will be moving upwards, following the projectile motion with initial velocity. The maximum displacement (3.391 inches) will accordingly be obtained from projectile motion equation ($v_1^2/2g + x_s$). The first cycle of time shifted ($\delta t = 0.055\text{sec}$) in Figure 3 due to rope slack can also be calculated by

$$\delta t = \frac{2v}{g} \cdot \frac{1}{f} \left[1 - \frac{a \cos(-x_s/x_0)}{\pi} \right] \quad \text{Eq. 11}$$

this time shift will be accumulated for each cycle motion as indicated from figure 3.

The rope forces will be zero after including self-weight of lifted object in Figure 3, and the maximum rope forces from the first approach is equal to $(x_0 + x_s) K = (3+1.81) \times 256/1.81 = 680 \text{ kips}$. It is the same with dynamic response of the rope forces shown in Figure 3, which is $(1.658 +1) W = 680 \text{ kips}$ after including self-weight. Because there is no input excitation, the maximum dynamic amplification factor is equal to 2.66 for both slack rope solution and elastic solution without slack rope.
Rope Dynamic Load with Harmonic Excitation

As discussed above, if the rope overcomes the stretch of the rope \((xs)\), the object will start to slack and travel in projectile motion. The object drops down and stretches the rope again with the time delay. This time delay may produce dynamic impact on the rope in an additive way with input excitation. For illustration, considering above the SDOF dynamic system with adding 4 percent damping, input a harmonic excitation with intensity \((p_0)\) in terms of gravity \((g)\) with excitation frequency \((f_{in})\). Thus, the excitation input at right side of Equation 2 can be substituted by \(p_0 \sin(2\pi f_{in} t)g\) as the function of time \((t)\). The nonlinear dynamic response can be easily solved by using RK direct integration (second approach).

For comparison, the dynamic response of the rope forces (excluding self-weight) for input harmonic excitation frequency at 0.7, 1.0, and 1.3 of natural frequency of the SDOF dynamic system are plotted in Figures 4, 5, and 6, respectively; the magnitude of the input harmonic excitations is 1.0g.
From dynamic response of the rope forces with three different input excitation frequencies, we found that

1. The dynamic amplification factors for slack rope are 5.5, 4.1, and 3.1, respectively (with self-weight included) for input frequencies $f_{in}$ at 0.7, 1.0, and 1.3 of system frequency $f$. The dynamic impacts on rope due to rope slack are significantly higher at the lower excitation frequency and gradually drop as input frequency increase.

2. The dynamic behavior for slack rope is different with elastic spring solution and no dynamic resonant response will occur for slack rope. In other words, dynamic impact for slack rope at resonant frequency is much lower than that of elastic-spring solution. Because half of the restoring force from the slack rope is constant gravity load, it will not be magnified as the spring force response in resonant condition.

3. As the input frequency gets higher from structure system frequency, or in other words, the frequency of the rope system is much lower than excitation frequencies. The dynamic amplification due to slack rope will be diminished; therefore, the elastic-spring solution becomes feasible by ignoring the nonlinear slack rope effects.

As discussed above, the object starts to toss up; the object is only subject to the constant gravity load. The time delay also prevents object dancing with harmonic excitation input in same tone. Low-frequency contents of excitation compared to the frequency of the rope system will most likely produce the impact effect on the rope due to slack rope. Because the rope length typically varies, it is very useful for investigators to judge the slack rope impact from the given frequency contents response spectra.

**DYNAMIC RESPONSE OF SLACK ROPE FOR EARTHQUAKE EXCIATION**

**Earthquake Time History Input**

Because the linear dynamic response spectra are not applicable for nonlinear slack rope system, due to lack of recorded time history database, the spectrum-compatible-artificial-synthesized time history becomes a very popular expedient solution for nonlinear dynamic time history analysis. The common method for synthesizing earthquake is that of superposing sinusoidal components with random phase angle. The amplitudes are determined from estimates of the spectral density function of ground motion.
They may vary in time or constant for duration of earthquake. SIMQKE is that kind of artificial synthesized time history program, developed by Gasparini and Vanmarck in 1976 [5]. It has been widely used over the last two decades. The main procedure of SIMQKE includes (i) deriving the spectral density function from the response spectrum, which is to be matched; (ii) adjusting the generated peak acceleration to match the target value; and (iii) adjusting the ordinates of the spectral density function to smoothen the match. The typical simulated time history by simulation is plotted in Figure 7.

Figure 7 - Typical Simulated Ground Acceleration Time Histories

The acceleration and velocity response spectra corresponding to three set of simulated time histories are plotted and compared to target design response spectra in Figures 8 and 9.

Figure 8 - Simulated Acceleration Spectra Comparison with Target Response Spectra
Dynamic Response of Slack Rope for Earthquake Input

As discussed above, nonlinear dynamic response for slack rope can be simplified and easily calculated without dealing with a large DOF nonlinear dynamic analysis. Considering a crane system that consists of a moving trolley truck with lifted hook riding on a crane bridge moving along the runway, a simplified 2DOF system can be established as described earlier for rope design. The lifted object weights and the filtered floor spectra at the runway are typically provided by manufactures, and the property of the rope and lifted load are specified as the same as previous section for SDOF system. The rope lengths vary according to site and project requirement specified by manufacturer’s provider. According to NOG 4153, moving trolley position on the crane bridge shall be considered at midspan, 1/4 span, and end span of crane bridge girders. For simplification, trolley position factors are considered by estimating equivalent stiffness and activated masses of the crane system from linear dynamic analysis for vertical mode. The equivalent stiffness and effective masses for crane system at different trolley position in this analysis are assumed here

<table>
<thead>
<tr>
<th>Effective Crane Weight (W1)</th>
<th>Equivalent Crane Stiffness (K1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trolley at midspan:</td>
<td>108 kips</td>
</tr>
<tr>
<td>Trolley at 1/4 span:</td>
<td>105 kips</td>
</tr>
<tr>
<td>Trolley at end span:</td>
<td>102 kips</td>
</tr>
</tbody>
</table>

where effective weights of the crane system are the activated vertical masses, and equivalent crane stiffnesses are estimated from the vertical deflection of crane at the trolley position produced by the lifted weight \(W2/\Delta_{\text{deflection}}\). For simplification, three rope lengths of the trolley truck at different positions are considered for high, mid, and low hook position. The frequencies of the crane system and the rope system are calculated by their definition, see Equation 7 and \(f_i=\omega_i/2\pi\). Both crane system and rope system have 4 percent damping. The natural frequency of the crane system combined with the rope system are calculated and listed in Table 1.
Table 1 - Natural Frequencies (Hz) of Crane System for Simplified 2DOF System

<table>
<thead>
<tr>
<th>Hook Position</th>
<th>Rope Freq., f2=6.721Hz</th>
<th>Crane Freq., Mode 1</th>
<th>Mode 2</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Hook</td>
<td>16.751</td>
<td>4.973</td>
<td>13.616</td>
<td>3.262</td>
<td>12.884</td>
</tr>
<tr>
<td>Mid Hook</td>
<td>18.795</td>
<td>5.4</td>
<td>16.114</td>
<td>3.359</td>
<td>15.501</td>
</tr>
<tr>
<td>Low Hook</td>
<td>21.594</td>
<td>5.76</td>
<td>19.336</td>
<td>3.43</td>
<td>18.824</td>
</tr>
</tbody>
</table>

The frequency contents of the vertical excitation of the earthquake are mostly located between 2Hz to 5Hz and the peak acceleration can be as high as 4.3g. Table 1 shows that the frequency variation of the rope system has fallen into earthquake frequency range, the significant impacts due to slack rope are expecting. Three sets of simulated dynamic time history response of rope forces with slack rope with 4 percent structural damping are compared with linear dynamic modal analysis and linear dynamic response spectra analysis for the 3-D crane system finite element analysis from GTStrudl computer modeling (not shown for simplicity). The results for comparison are shown in Table 2.

Table 2 - Rope Forces (g’s) without Slack Rope

<table>
<thead>
<tr>
<th>Position</th>
<th>Hook</th>
<th>Rope Length (ft)</th>
<th>Time History Analysis</th>
<th>Modal Analysis</th>
<th>GTStrudl</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>TH(Set-1)</td>
<td>TH(Set-2)</td>
<td>TH(Set-3)</td>
</tr>
<tr>
<td>Trolley at Midspan</td>
<td>High Hook</td>
<td>8.5</td>
<td>3.878</td>
<td>3.917</td>
<td>3.822</td>
</tr>
<tr>
<td></td>
<td>Mid Hook</td>
<td>30</td>
<td>6.144</td>
<td>6.249</td>
<td>5.788</td>
</tr>
<tr>
<td></td>
<td>Low Hook</td>
<td>71</td>
<td>4.698</td>
<td>4.648</td>
<td>5.59</td>
</tr>
<tr>
<td>Trolley at 1/4 Span</td>
<td>High Hook</td>
<td>8.5</td>
<td>3.179</td>
<td>3.382</td>
<td>3.193</td>
</tr>
<tr>
<td></td>
<td>Mid Hook</td>
<td>30</td>
<td>6.001</td>
<td>6.108</td>
<td>5.716</td>
</tr>
<tr>
<td></td>
<td>Low Hook</td>
<td>71</td>
<td>4.884</td>
<td>4.789</td>
<td>5.542</td>
</tr>
<tr>
<td>Trolley at End Span</td>
<td>High Hook</td>
<td>8.5</td>
<td>3.172</td>
<td>3.059</td>
<td>3.32</td>
</tr>
<tr>
<td></td>
<td>Mid Hook</td>
<td>30</td>
<td>5.534</td>
<td>5.465</td>
<td>5.874</td>
</tr>
<tr>
<td></td>
<td>Low Hook</td>
<td>71</td>
<td>5.143</td>
<td>4.934</td>
<td>5.515</td>
</tr>
</tbody>
</table>

From the table, the results show that the linear dynamic response of the rope force from modal analysis for simplified 2DOF system has less than 1 percent that compared to the response spectra analysis for multi-DOF analysis from GTStrudl results. Therefore, the simplified 2DOF system is accurate to represent crane system for rope analysis. For comparison, three sets of linear time history analysis from the above simulated earthquake are also compared in Table 1. The results show that linear time history analysis has the reasonable accuracy comparing with corresponding response spectra analysis. Because the 2DOF dynamic system has been proven to be effective and accurate in nonlinear rope slack analysis, direct integration RK method can be easily applied to 2DOF system for nonlinear time history analysis of the slack rope. The results for nonlinear time history analysis for slack rope dynamic response with various rope lengths are shown in Figures 9 and 10 (self weights are included in rope forces) for 4 percent and 7 percent damping of the rope system.
The results from diagrams above show that the slack rope impacts are not significant at the beginning for very short rope length, but quickly rise to the peak as the rope length increasing. The impact load due to slack rope becomes very significant, and then gradually drops as the rope length continuously increases. In other words, under severe vertical earthquake excitation (here, 4.3g peak acceleration), the lifted weight bundled with the crane bridge together for short rope lengths. As the rope length reaches a certain length, the rope starts to slack, and the lifted object gives a significant impact on the rope. As the rope length increases, lifted weight is gradually isolated from the earthquake due to longer rope lengths like soft-spring. The slack rope impact is reduced accordingly, and is matched to the elastic solution without
slack rope. The impact loads due to slack rope are also much more severe than that of elastic spring expectation. These dynamic characters for slack rope are quite different with that of elastic spring solution with various rope lengths. In the elastic spring solution, the impact load peaks at midhigh hook position rather than high hook position from nonlinear slack rope solution. Increasing the stiffness of the crane bridge, the rope length that results in slack rope impact may increase, as well. Adding damping into the rope system always helps the rope forces.

THE BRAKE SLIP FOR IMPACT OF SLACK ROPE

Brake Slip Mathematic Model
The above discussion shows that, at the certain length of the rope, the rope slack may introduce a large impact load on the rope under severe earthquake excitation. Furthermore, it may result in a large impact force on crane bridge structures. This impact may be very expensive to design the rope, crane, support structures, and other attachment components. To mitigate this impact load, one of slip-brake device was proposed by the manufacturer. The mechanism of the slip-brake is that the rope force reaches a certain threshold and the brake starts to slip. The rope force bears the constant brake resisting forces until the brake stops slips, and the object starts to rebound upwards. Based on this mechanism, the nonlinear restoring force in Equation 5 for slack rope may be modified by introducing new variable \( y \),

\[
F_2(x,y) = \begin{cases} 
-g & \text{if } x < x_s \\
-g & \text{if } y < g / F_y \\
\alpha \cdot \omega^2 \cdot x + (1 - \alpha) \cdot F_y \cdot y & \text{otherwise}
\end{cases}
\]

where the brake resisting force \( F_y \) is typically specified by the brake manufacturer. The relative displacement \( (x) \) between lifted load and crane is defined \( (x_2 - x_1) \) in Equations 1 and 2. The radian frequency \( (\omega) \) has the same definition as \( (\omega_2) \) in equation 7. The \( \alpha \) is the hardening slope for the constant brake forces. The new variable \( (y) \) can be solved by combining the following equation \( H(v,y) \), by using bilinear model proposed by Asano and Iwan [6] in 1984

\[
H(v,y) = \begin{cases} 
\frac{\omega^2}{F_y} \cdot v & \text{if } y < g / F_y \\
\frac{\omega^2}{F_y} \cdot [ -\text{unit}(v) \cdot \text{unit}(y-1) - \text{unit}(-v) \cdot \text{unit}(-y-1) ] & \text{otherwise}
\end{cases}
\]

where the unit function \( \text{unit}(x) \) is equal to unity when \( x \geq 0 \) and null when \( x < 0 \). The relative velocity \( (v) \) which \( v = \dot{x}_2 - \dot{x}_1 \), has the same definition as defined in Equations 1 and 2.
Dynamic Response for Brake Slip with Slack Rope

Considering a SDOF system with same parameters as previous sections, assume brake force \( (F_y) \) is twice of the gravity load \((2g)\) and harmonic excitation input. The intensity harmonic excitation is three times that of that gravity load \((3g)\), and harmonic excitation frequency is 0.7 of the structural natural frequency of the rope system. No hardening is assumed for brake slip \((\alpha = 0)\). The relationship of brake force with slip displacement is plotted in Figure 11. If the rope force is larger than twice that of gravity \((2g)\), the brake starts to slip on the other side, the rope overcomes the self-weight of the lifted load, and then the rope begins to slack. Because the slip displacement cannot be recovered, the dynamic response of displacement of the lifted object due to slack-slip motion will be constantly increased as shown in Figures 11 and 12.

The comparison of dynamic response of rope forces (without self-weight) and displacement for the slack-slip, slack, and no slack are shown in Figures 12 and 13. The results from diagrams show that the brake slip device for slip-slip successfully limits the rope force with the target brake slip force. But the price should be paid for constantly increasing displacement of lifted object by slipping the brake device rather than tossing the lifted object into the air and hitting the rope. Because the slip displacement is predictable by solving nonlinear dynamic equations, it is rather easy and economic to find the availability of the environment than design enormous impact load due to slack rope, particularly, for high hook position where the severe slack rope impact most likely occurs.

![Figure 12 - Dynamic Response Comparisons of Rope Forces](image)

![Figure 13 - Dynamic Response Comparisons of Displacement](image)
CONCLUSION AND DISCUSSION

Simplified analysis by using 2DOF dynamic system instead of the complex trolley-crane system proves to be very efficient and accurate in rope force study for vertical earthquake excitation. Further, it is very easy and affordable to apply nonlinear restoring force function for nonlinear time history analysis. Elastic linear spring analysis for the rope design is significantly underestimate dynamic impact load on rope due to rope slack especially at short rope length range. Therefore, nonlinear dynamic response for various rope lengths is necessary if the slack rope occurs. For the excitation frequency contents lower than that of lifted load with rope, the dynamic impact load for slack rope is significant and becomes less significant as the excitation frequency increase gradually; in other words, the longer rope length has the less dynamic impact load. No resonant will occur.

The brake-slip device can limit impact forces from slack rope and prevent the rope failure from impact load. Proposed mathematic model for slip and slack of the rope reveals the mechanism of the slip-sack of the rope under severe earthquake excitation and can also predict the maximum slip displacement for the prescribed seismic load.

REFERENCES